Model of intermittency in magnetohydrodynamic turbulence

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We extend to the magnetohydrodynamic (MHD) case a recent model of intermittency due to She and Lévêque [Phys. Rev. Lett. **72**, 336 (1994)]. The model that we develop in the framework of the Iroshnikov-Kraichnan theory of MHD turbulence depends on two parameters that are linked to anomalous scaling laws of dissipative structures when their characteristic scale $l \to 0$. A brief comparison with published data stemming from spacecraft observations within the solar wind [L. Burlaga, J. Geophys. Res. **96**, 5847 (1991)] shows that it is a workable model and that within the framework of the model, dissipative structures in MHD turbulence are sheetlike, as observed in recent numerical simulations in three dimensions at moderate Reynolds numbers.

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I. INTRODUCTION

Intermittency in turbulent flows has been observed both in the laboratory and in numerical simulations. This phenomenon reflects the spatial scarcity of intense small-scale dissipative structures and is believed to be responsible for the departure from a pure Kolmogorov (1941) scaling for the structure functions $S_p = \langle [v(x+l)-v(x)]^p \rangle$ of the velocity field, a departure more strongly felt at higher p. The ensuing breaking of global scale invariance has led to the construction of several models of intermittent flows, most of which are parameter dependent (see, for example, [1] and references therein). In the solar wind [2, 3], which is believed to be in a state of Alfvénic turbulence [4], intermittency has been inferred as well through, for example, the computation of structure functions. A recent parameter-free model due to She and Lévêque [5] (hereafter referred to as SL) currently attracts much attention, in particular because of its excellent (better than 1%) agreement with the laboratory data of [6] and also because it reflects an underlying logarithmic Poisson statistics [7, 8] and its associated rare events. We propose here to extend this model to the magnetohydrodynamic (MHD) case in the framework of the Iroshnikov-Kraichnan theory for MHD turbulence [9] (hereafter referred to as IK) and confront it to observational data within the solar wind of [2].

II. THE SL MODEL OF INTERMITTENCY

Writing the (say, longitudinal) velocity difference across a length l as $\langle \delta v_l \rangle = \langle v(x+l)-v(x) \rangle$, its p moments scale as $\langle \delta v_l^p \rangle \sim l^{\zeta_p}$ with, for the classical Kolmogorov-1941 (or K41) law [10] $\zeta_p^{(K)} = p/3$; in particular, for $p = 2$, the energy spectrum reads $E(k) \sim \bar{\epsilon}^{2/3} k^{-5/3}$, where $\bar{\epsilon}$ is the average rate of energy dissipation. However, both experimental and numerical data disagree with the Kolmogorov law (K41), a fact attributed to intermittency [11]. In fully developed turbulence, fluctuations of energy transfer to small scales are presumably present and are deemed responsible for the intermittency of the flow. Writing now, for the typical dissipation rate at scale l, $\langle \epsilon_l^p \rangle \sim l^{\tau_p}$, and using the Kolmogorov (1962) refined similarity (KRS) hypothesis [12] $\epsilon_l \sim \delta v_l^3 / l$, we immediately obtain $\zeta_p = p/3 + \tau_{p/3}$ with $\zeta_0 = 0$ and $\zeta_3 = 1$ and hence $\tau_0 = 0$ and $\tau_1 = 0$. The latter is an exact result for steady fluids, stemming from the conservation law of energy with isotropy, incompressibility, and homogeneity assumed. The KRS hypothesis above, implying a correlation between the local dissipation rate ϵ_l and local velocity fluctuations δv_l, may be weaker in the inertial range than in the viscous range [13] and should be tested, e.g., through careful evaluation of experimental and numerical data.

Numerous models have been derived to predict the functional variation of the scaling exponents ζ_p of the velocity structure functions with p. The successful assumption made by SL is to write a scaling law for the successive powers of the energy dissipation at scale l. Defining $\epsilon_l^{(p+1)} = \langle \epsilon_l^{p+2} \rangle / \langle \epsilon_l^{p+1} \rangle$, the assumed scale dependence reads

$$\epsilon_l^{(p+1)} = A_p \epsilon_l^{(p)^\beta} \epsilon_l^{(\infty)^{1-\beta}} , \tag{1}$$

where $0 < \beta < 1$; in fact, in the SL model, $\beta \equiv 2/3$ and $\epsilon_l^{(\infty)} \sim v_0^2 / t_l$ is an estimate of the maximum amount of energy that can be dissipated in the most intermittent structures in a time $t_l \sim l^{2/3}$, in accordance with standard K41 phenomenology. This is equivalent to assuming that the divergence of the energy flux as $l \to 0$ follows a 2/3 anomalous scaling law. This 2/3 law in turn leads to $\tau_p = -2p/3 + C_0 + f(p)$, $C_0 \equiv 2$ being interpreted as the codimension of dissipative structures, taken to be filaments (or tubes) in dimension three, and $f(p) = -C_0 \beta^p$. Once C_0 and β are determined on physical grounds, the resulting SL model is parameter-free. In Eq. (1), β measures the degree of efficiency of energy transfer from scale

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to scale. Defining $\pi_l = \epsilon_l/\epsilon_l^{(\infty)}$ as a normalized transfer rate, Eq. (1) can also be written

$$\langle \pi_l^p \rangle = B_p \langle \pi_l \rangle^{\frac{1-\beta^p}{1-\beta}} \quad ,$$

leading to a $\log_\beta$ Poisson statistics for $B_p = 1\ \forall p$ (see [7, 14]). For $\beta = 1$, a hierarchical relationship holds, namely, $\langle \pi_l^p \rangle = \langle \pi_l \rangle^p$ corresponding to the Kolmogorov law, whereas for $\beta = 0$ one finds $\langle \pi_l^p \rangle = \langle \pi_l \rangle\ \forall p$, an extreme case of intermittency since it implies that all the dissipation is concentrated in one single structure of characteristic transfer rate $\epsilon_l = \epsilon_l^{(\infty)}$.

In fact, we can recast the SL model in the framework of a two-parameter model, a formulation that can also be found in [7, 8]. Indeed, the scaling of the most dissipative features can be left open (i.e., $t_l \sim l^x$ with $x > 0$) and so can the codimension C_0 of the intermittent structures. We thus obtain (see also [7, 8]) $\tau_p = -xp + C_0 + f(p)$ and $f(p) = -C_0\beta^p$, leading to

$$\zeta_p = \frac{p}{3}(1-x) + C_0[1-\beta^{p/3}] \quad , \quad C_0 = \frac{x}{1-\beta} \quad . \qquad (2)$$

Writing $\zeta_6 = 2 - \mu_6$ and $\zeta_9 = 3 - \mu_9$ we have $\mu_6 = x^2/C_0 = x(1-\beta)$ and $\mu_9 = x(2-\beta-\beta^2)$ so that their ratio $r_{9,6} = \mu_9/\mu_6$ is independent of x. Thus the experimental measurements of the ζ_6 and ζ_9 exponents lead directly to the determination of the two parameters of the model (with $0 < \beta < 1$) since

$$\beta = -2 + r_{9,6}, \quad x = \mu_6/(1-\beta) \quad . \qquad (3)$$

Note that we have $2 < r_{9,6} < 3$; when $\beta = 1$ corresponding to the nonintermittent case, $\mu_6 \equiv 0$ and $x \equiv 0$ [15].

From the experimental determination of ζ_p in [6], we obtain $\beta \sim 0.68$ and $x \sim 0.69$, close to the original values in the SL model. Taking now the published results of three-dimensional numerical simulations of homogeneous turbulent flows in [16], we find $\beta \sim 0.76$ and $x \sim 0.71$. The discrepancies between the two sets of values either can be due to the fact that the numerical data is still somewhat unresolved, the Taylor Reynolds number being quite low compared to that of experimental data, or can be linked to the extended self-similarity (ESS) hypothesis [6], whereby self-similarity is extended to the viscous subrange by plotting structure functions against each other, e.g., in terms of ζ_3 homogeneous to a length scale, as opposed to using the variable l in the abscissa. This hypothesis is not yet fully tested, in particular for large differences between the orders of the structure functions [17], although it is known to work in the framework of scalar models of turbulence [14] including in the MHD case [18]. In order to test whether this leads to a better agreement with SL, one could introduce as well the ESS hypothesis into the analysis of the numerical data in [16]. It should also be noted that in shell models with hyperviscosity (whereby the dissipative Laplacian operator written in Fourier space k^2 is replaced by $k^{2+\alpha}$ with $\alpha > 0$), it has been shown [19] that $\zeta_3 \neq 1$, but that the universality in scaling is recovered when dealing with a normalized exponent $\tilde{\zeta}_p = \zeta_p/\zeta_3$ (in relation in fact to the ESS hypothesis).

III. COUPLING TO A MAGNETIC FIELD

We now proceed to extend the SL model to the case of the coupling between the velocity and the magnetic field. The incompressible MHD equations using the Elsässer variables $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$ read

$$\frac{\partial \mathbf{z}^\pm}{\partial t} + \mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm = -\nabla p^* + \nu^+ \nabla^2 \mathbf{z}^\pm + \nu^- \nabla^2 \mathbf{z}^\mp \qquad (4)$$

and $\nabla \cdot \mathbf{z}^\pm = 0$, with $p^* = p/\rho + b^2/2$ the total pressure, $\mathbf{v}$ the velocity, and $\mathbf{b}$ the magnetic field (in units of the Alfvén velocity); $2\nu^\pm = \nu \pm \eta$, where ν and η are the viscosity and the magnetic diffusivity. In the IK theoretical framework, it has been argued [9] that in MHD turbulence, nonlinear transfer to small scales are hampered by the fact that $\mathbf{z}^\pm$ eddies only interact when they meet, as they travel in opposite directions along the lines of a large-scale quasiuniform magnetic field B_0. This reduces drastically the level of energetic transfer to small scales [20] so that the average energy transfer time is now $\tau_{\rm tr} = \tau_{\rm nl} R_{\rm eff}$, where

$$R_{\rm eff} = (\tau_{\rm nl}/\tau_A)^a$$

measures the efficiency of the nonlinear interactions, $\tau_{\rm nl} = \ell/z$ is the eddy turnover time (where $z^+ \sim z^- \sim z$ in the uncorrelated case; see [21]), $\tau_A = \ell/B_0$ is the Alfvén time, and a is an arbitrary power law. It is easy to show that the flux scales as $\epsilon_a \sim z^g/\ell B_0^a$ with

$$g = a + 3 \quad ;$$

the ensuing energy spectra read $E_a(k) = (\bar{\epsilon}_a B_0^a)^{2/g} k^{-(1+2/g)}$. In particular, in the IK theory, $a = 1$; this leads to an isotropic energy spectrum (either for $E^T = E^V + E^M$, where E^V and E^M are the kinetic and the magnetic energies, or for $E^\pm$ corresponding to the $z^\pm$ variables)

$$E(k) \sim (\bar{\epsilon} B_0)^{1/2}\ k^{-3/2} \quad .$$

This IK spectrum assumes that the correlations between the velocity and the magnetic field, as measured, for example, by the coefficient $\rho = \frac{2\langle \mathbf{v}\cdot\mathbf{b} \rangle}{\langle v^2 + b^2 \rangle}$, are null [22]. There are many numerical studies of MHD turbulence showing the plausibility of the IK scaling [23], as well as other consequences of such reduced transfer; for example, the fractal dimension of isodensity surfaces of the current density in MHD turbulence (and of vorticity) has been estimated following [24], but taking Alfvén waves into account [25]. Numerical simulations in MHD turbulence on a two-dimensional grid of 2048^2 points [25] and in the fluid case in three dimensions on a grid of 240^3 points with good temporal statistics [26] both appear to agree with such estimates (although the error bars are large).

The MHD equations have the same type of scale invariance as the Navier-Stokes equations (namely, $\ell \to \lambda\ell$, $U \to \lambda^h U$, and $t = \ell/U$), as well as energy conservation; both types of constraints act on nonlinear interactions; power laws in ℓ/ℓ_0, where ℓ_0 is the energy-containing

scale, arise as natural solutions to the dynamical equations, broken only in the dissipative range. The relationship equivalent to the Kolmogorov refined similarity hypothesis for MHD turbulence reads

$$\epsilon_l \sim \delta z_l^g / \ell B_0^a \ ,$$

leading to $\zeta_p = p/g + \tau_{p/g}$. Assuming

$$\epsilon_l^{(p+1)} = A_p^{(B)} \epsilon_l^{(p)^{\beta_B}} \epsilon_l^{(\infty)^{1-\beta_B}} \tag{5}$$

with $0 < \beta_B < 1$, together with $t_l \sim l^{x_B}$, we obtain $\tau_p = -x_B p + C_0^B + f(p) = -x_B p + C_0^B(1-\beta_B^p)$ as before. The analysis follows straightforwardly as in the SL case and gives

$$\zeta_p = \frac{p}{g}(1-x_B) + C_0^B(1-\beta_B^{p/g}) \ , \quad C_0^B = \frac{x_B}{1-\beta_B} \ , \tag{6}$$

where we have used, as in the fluid case, $\tau_0 = 0$ and $\tau_1 = 0$ (since the mean flux $\bar{\epsilon}$ is constant by definition) for all g; thus, in the general case $\zeta_{p=g} \equiv 1$. For the IK phenomenology ($a = 1$) to which we now restrict the analysis, $g = 4$ [25, 27], leading to $\zeta_4 \equiv 1$. The model of intermittency in MHD turbulence that is proposed here in the framework of the IK theory thus reads

$$\zeta_p = \frac{p}{4}(1-x_B) + C_0^B[1-\beta_B^{p/4}] \ , \quad C_0^B = \frac{x_B}{1-\beta_B} \ . \tag{7}$$

The scaling of structure functions depends on the nonlinearities of the system and thus differs for Navier-Stokes and MHD turbulence. The relationship $\zeta_g \equiv 1$, on the other hand, is of a different nature. For K41, it can be demonstrated from the primitive equations using homogeneity, isotropy, incompressibility, and stationarity. In the MHD case, such a demonstration is lacking. It has been shown in the context of shell models [14, 19] in the fluid case that one may have $\zeta_3 \neq 1$ and yet recover scaling laws for the structure functions in good agreement with the SL model for the *relative* exponents $\tilde{\zeta} = \zeta_p/\zeta_3$. Furthermore, in [28], in the context of the analysis of direct numerical simulations of two-dimensional Navier-Stokes turbulence, it is shown that the relative exponents $\tilde{\zeta}_p$ are to be used in agreement with the ESS hypothesis, in part because they are better evaluated experimentally since they give rise to a more extended power-law range. In that context, Eqs. (7) and (6) may apply either to absolute exponents or to relative ones ζ_p/ζ_g.

Defining now $\zeta_8 = 2 - \mu_8^B = 2 - x_B(1-\beta_B)$ and $\zeta_{12} = 3 - \mu_{12}^B = 3 - x_B(2 - \beta_B - \beta_B{}^2)$, with $r_B = \mu_{12}^B/\mu_8^B$, we can again solve simply for x_B and β_B in MHD turbulence with

$$\beta_B = -2 + r_B \ , \quad x_B = \mu_8^B/(1-\beta_B) \ , \tag{8}$$

given reliable data up to the 12th order of the structure functions.

Of the two-parameter (x_B, C_0^B) models in Eq. (7), one stands out, which we coin "standard"; it obtains assuming $x_B^{(S)} \equiv 1/2$, in agreement with the scaling used in deriving the IK spectrum, and assuming a fixed and integer codimension of dissipative structures, which in MHD turbulence are close to sheets [29], in line with the phenomenological analysis made in the SL model for the fluid equations; thus $C_0^{B,S} \equiv 1$, and we again have $x_B^{(S)} = \beta_B^{(S)}$. In the standard case with no free parameter, one then obtains

$$\zeta_p^{(S)} = \frac{p}{8} + 1 - 1/2^{p/4} \ , \tag{9}$$

with dissipative structures being sheetlike. For the energy spectrum ($p = 2$), the standard model gives $E(k)^{(S)} \sim k^{-\frac{3}{2}-0.04}$, a correction slightly larger than in the SL model for fluids. Equation (9) has been derived independently in [30]. We wish to remark that in MHD turbulence, it is found that the dissipative layers coincide spatially with current and vorticity sheets [29], whereas vortex filaments themselves (in the fluid case) are not dissipative and thus exist for times long compared to the eddy turnover time and are sheathed by dissipation.

Note that if $C_0 = x/(1-\beta)$ is the codimension of dissipative structures, it follows that $C_0 \leq D$, where D is the dimension of space. Assuming that the parameter x is known (for example, being determined from phenomenology), this in turn gives a condition on the second parameter of the model, namely, $\beta \leq 1 - x/D$. With superscripts denoting dimensionality of space and subscripts referring to either the fluid or the MHD case and with the "standard" values $x_{\rm NS}^{(S)} = 2/3$ and $x_B^{(S)} = 1/2$, this condition reads $0 < \beta \leq \bar{\beta}$ with in the fluid case $\bar{\beta}_{\rm NS}^{(3D)} = 7/9$ and $\bar{\beta}_{\rm NS}^{(2D)} = 2/3$, whereas in MHD turbulence we obtain $\bar{\beta}_B^{(3D)} = 5/6$ and $\bar{\beta}_B^{(2D)} = 3/4$. Note that the condition in MHD turbulence is less restrictive than for Navier-Stokes turbulence.

IV. COMPARISON WITH SOLAR WIND DATA

We now confront the proposed model with published solar wind data, that of the Voyager spacecraft [2] at 8.5 astronomical units (A.U.). Following the analysis in [14, 19], we use the relative exponents ζ_p/ζ_4 for the analysis of the solar wind data, compatible with IK phenomenology and obtain from Eqs. (8) $\beta_B = 0.45$ and $x_B = 0.52$ (hence $C_0^B = 0.95$). Taking into account the error bars in the data, the ranges for the two parameters of the MHD turbulence model are $0.32 \leq \beta_B \leq 0.52$ and $0.50 \leq x_B \leq 0.53$, leading to $0.74 \leq C_0 \leq 1.09$. Note that the computed value of x_B is close to that stemming from the IK theory; but the second relationship of the standard SL model (namely, $x_B = \beta_B$) does not appear to be fulfilled when taking into account the lowest values compatible with the error bars. A similar discrepancy was already noticed in the context of shell models for fluids [14], where $x = \beta$ only when a second invariant of the model equations—kinetic helicity—obtains, besides the energy, a result that could be linked to the dynamical importance of invariants. It should be noted that imposing both the value of x and the equality $x = \beta$ exactly determines the codimension of dissipative structures once the anomalous scaling of the temporal evolution ($t_\ell \sim \ell^x$)

is fixed by dimensional constraints and/or phenomenology; this indeed leads to filaments in three-dimensional Navier-Stokes turbulence and presumably sheets in MHD turbulence. Note that several types of dissipative structures may arise in the dynamical evolution of the flow, leading to some sort of multifractality. If we pursue this interpretation of the parameter C_0 as being the codimension of the dissipative structures, we see that these structures in the solar wind at 8.5 A.U. are of dimension ~ 2.11, i.e., sheetlike; similar structures have been observed in recent three-dimensional simulations of MHD turbulence [29]. The evaluation of this dimension of dissipative structures is to be compared with that stemming from a different model of intermittency [18] based on a binomial process in which the open parameter relates to an asymmetry in energy transfer to small eddies; this dimension is found to be ~ 2.9, i.e., very close to isotropic structures. However, we should note that the determination of the two parameters of the model using Eqs. (8) is very sensitive to the precise evaluation of the anomalous exponents of the structure functions.

For high values of the structure function exponent p, the scaling $\zeta_p = f(p)$ in [2] appears linear with a slope ~ 0.15, compatible with the linear part of the standard IK law $\zeta_p \sim p/8$. Similarly, examining the numerical data for three-dimensional (3D) fluid flows in [16] at high p, the scaling appears again linear with now a slope ~ 0.1, to be compared with $\zeta_p = p/9$ for the linear part of the SL model. Note that at high p, we get $\delta z_l^p \sim \delta z_{\max}^p$ so that the scaling of high-order structure functions are in fact the scaling for the suprema of the variables.

Far from the sun, as for the data just examined here from Voyager in [2], the turbulence has had time to fully develop. On the other hand, in the data from Helios [3] at distances between 0.3 and 1 A.U., the flow may very well be dominated by waves, with little nonlinear transfer to small scales. As shown in [3], the scaling of ζ_p is nonlinear, with a slow growth of the ζ_p exponents with p. Although parameters for the model could be adjusted to the data, the model presented in this paper may not necessarily apply because of a lack of substantial nonlinear transfer, as well as strong dissipation of outward propagating $\mathbf{z}^{\pm}$ waves as suggested in [31].

V. DISCUSSION

A two-parameter intermittency model for MHD turbulence is proposed along the lines of the one derived in [5, 7, 8] in the fluid case. It results in the prediction of the variation of structure functions of order p with p given in Eq. (7). The two parameters are related respectively to the anomalous scaling of the characteristic time of the most intermittent structures and to their codimension. The essential assumption of the model, following [5], is linked to the existence of a hierarchy of successive moments of energy dissipation given in Eq. (5). The model is confronted with the observed anomalous exponents of structure functions in the solar wind far from the sun [2]; relatively good agreement obtains with the standard model of Eq. (9).

An open question here concerns the role of the other two invariants of the MHD turbulence equations, namely, the cross correlation $E^C = \langle \mathbf{v} \cdot \mathbf{b} \rangle$ and the magnetic helicity $H^M = \langle \mathbf{A} \cdot \mathbf{b} \rangle$ [32] (or in two dimensions $A^M = \langle \mathbf{A} \cdot \mathbf{A} \rangle$), where $\mathbf{A}$ is the magnetic potential. As noted before, a lack of universality in the exponents β and x was obtained in [14] in the context of shell models of turbulent transfer: indeed, a wide variety of values for such exponents is found, with a sharp transition with $x = \beta$ *only* when the model fulfills the invariance (in the inviscid case) of both the kinetic energy *and* the kinetic helicity $H^V = \langle \mathbf{u} \cdot (\nabla \times \mathbf{u}) \rangle$. Whether the $x = \beta$ relationship, within the framework of shell models, holds only when $\partial_t H^V = 0$ is fulfilled could be checked further by computing the anomalous exponents x and β of shell models with a different exponentiation for the discretization of wave numbers for which the dual invariance property holds for a different value of the coupling parameter in the shell model equations, as stated in [33].

In the fluid case, is the fact that in the SL model $\beta = 2/3$ (and thus $x = \beta$) a mere symptom of the condition that the cascade of energy (although in opposite directions in two and in three dimensions) nevertheless is ruled by the same SL-type constraint so that in three dimensions in fact the flow achieves the *minimum* intermittency compatible with local 2D dynamics? Recall that the Biot-Savart swirling flow induced by the strong local vorticity of an isolated filament is quasi-two-dimensional and is found in numerical simulations to be ~ 1. It could itself undergo locally an inverse cascade of energy initiating a flow at larger scales. The filament itself having a large aspect ratio, this large-scale flow could be felt on distances of the order of some fraction of the integral scale. If we take as a diagnostic of intermittency the magnitude of β (the farther from unity, the more intermittent the flow), then we note that in the 3D case β takes the maximal value allowed by the 2D case or, in other words, *the minimal* intermittency compatible with the local quasi-2D dynamics. This remark is to be connected with the fact that, following [34], a minimization principle of energy dissipation taking into account the invariance of helicity leads to Beltrami flows for which, of course, the Lamb vector $\mathbf{u} \times \boldsymbol{\omega}$ (where ω is the vorticity) is null (within pressure terms) and no intermittency can develop [recall that, using Schwartz inequality, the invariance of kinetic (magnetic) helicity indeed provides a lower bound to kinetic (magnetic) energy]. Although the above argument is speculative, it allows for an understanding of the role played by the conservation properties related to vorticity. These constraints are strong since, for isolated filaments, they are local conservation laws. However, such a helical structure is unstable unless the nonzero axial flow w satisfies $w = C(\psi)$, where ψ is the stream function associated with the 2D swirling flow (see [35]). In fact, whereas in the initial phase of evolution of a turbulent flow energy goes mostly to small scales, it could be that in the later stages in a quasisteady flow, energy goes both ways with a small resultant energy transfer to small scales [36]. This might be where helicity conservation enters in a phenomenological description of intermittent mode coupling in turbulence; however, only vorticity of the order of 10% is contained in the filaments

on which helicity (and circulation) is invariant. This type of argument, moreover, does not take into account the interactions between filaments, or the interaction of the filament with the turbulence bath containing most of the energy, and playing the role of a heat bath by analogy with thermodynamics.

On the other hand, in MHD turbulence with $x_B = 1/2$, we have $\bar{\beta}_B^{(3D)} > \bar{\beta}_B^{(2D)} > x_B$ (see the end of Sec. III); furthermore, the dissipative structures are quite unstable, contrary to their fluid counterpart. They keep evolving, for example, through X-point reconnection; this gives rise to the possibility of further intermittency because of small-scale activity linked, for example, to the tearing mode, presumably counterbalanced by the ordering due to an inverse cascade of either A^M or H^M. To further understand the role of the conservation of the topological MHD turbulence invariants H^M and A^M, it might be useful to generalize the MHD turbulence shell models (see, e.g., [37]) in two ways: (i) similarly to what is done for fluids by introducing nonlinear coupling between a wider range of shells, whereby allowing for a definition of magnetic (and kinetic) helicity in order to study the effect of its conservation on intermittency, and (ii) using complex variables, it should be possible to introduce the Alfvén coupling leading to the IK (instead of the Kolmogorov law) spectrum.

The analysis of experimental, numerical, and geophysical data far from the Sun has led to a scaling for the characteristic time t_l close to the one expected from dimensional analysis (the Kolmogorov law in fluid turbulence and the IK theory in MHD). This would be worth checking with numerical simulations of MHD flows, for example, in the context of a model of intermittent heating of the solar corona [38]. A three-dimensional numerical simulation of forced MHD turbulence in the homogeneous case can also be performed, although the spatial resolution attainable by present-day computations is moderate. However, when using algorithms with a reduced set of wave numbers, one finds that at high Reynolds number the intermittency corrections to the Kolmogorov law disappear *within the inertial range* [39]. Does the same result hold in MHD turbulence? This suggests that one should (a) perform in MHD turbulence a computation using a lean method (in terms of memory and CPU-time), possibly in the spirit of [40], allowing as well the investigation of the role of correlations between the velocity and the magnetic field, and (b) perform a scale-dependent intermittency correction analysis for solar wind data, as well as logarithmic Poisson statistics, compatible with the SL analysis both in the fluid and in the MHD case. In the same vein, one should check the ESS hypothesis directly and in particular perform the scaling analysis of structure functions with ζ_4 (as opposed to ℓ) as the independent variable.

Note added. A paper [30] has been brought to our attention, after completion of this work, that derives independently the extension of the SL model to MHD in the weakly correlated case—as given in Eq. (9)—assuming both standard IK temporal scaling and the development of current sheets.

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